Vector Algebra

Question 1.

A unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is

(a) $\frac{4\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{26}}$ (b) $\frac{2\hat{i} - 6\hat{j} - \hat{k}}{7}$

(c) $\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$ (d) $\frac{2\hat{i} - 3\hat{j} - 6\hat{k}}{7}$

Answer:

(c) $\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

Question 2.

The area of parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$ is

(a) 10√6

(b) 5√6

(c) 10√3

(d) 5√3

Answer:

(b) 5√6

Question 3.

If AB × AC = $2\hat{i} - 4\hat{j} + 4\hat{k}$, then the are of ΔABC is

(a) 3 sq. units

(b) 4 sq. units

(c) 16 sq. units

(d) 9 sq. units

Answer:

(a) 3 sq. units

Question 4.

A vector of magnitude 5 and perpendicular to $(\hat{i}-2\hat{j}+\hat{k})$ and $(2\hat{i}+\hat{j}-3\hat{k})$ is

(a) $\frac{5\sqrt{3}}{3}(\hat{i}+\hat{j}+\hat{k})$ (b) $\frac{5\sqrt{3}}{3}(\hat{i}+\hat{j}-\hat{k})$

(c) $\frac{5\sqrt{3}}{3}(\hat{i}-\hat{j}+\hat{k})$ (d) $\frac{5\sqrt{3}}{3}(-\hat{i}+\hat{j}+\hat{k})$

Answer:

(a) $\frac{5\sqrt{3}}{3}(\hat{i}+\hat{j}+\hat{k})$

Question 5.

$|a \times b|^2 + |a.b|^2 = 144$ and $|a| = 4$, then $|b|$ is equal to

(a) 12

(b) 3

(c) 8

(d) 4

Answer:

(b) 3

Question 6.

If $|a \times b| = 4$ and $|a.b| = 2$, then $|a|^2 |b|^2$ is equal to

(a) 2

(b) 6

(c) 8

(d) 20

Answer:

(d) 20

Question 7.

If $\vec{a}=(\hat{i}+\hat{j}+\hat{k}), \vec{a}.\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is

(a) $i-\hat{j}+\hat{k}$ (b) $2\hat{j}-\hat{k}$ (c) $\hat{i}$ (d) $2\hat{i}$

Answer:

(c) $\hat{i}$

Question 8.

The two vectors a = $2\hat{i}+\hat{j}+3\hat{k}$ and b = 4 \hat{i}-\lambda \hat{j}+6 \hat{k} ae parallel, if λ is equal to

(a) 2
(b) -3
(c) 3
(d) 2
Answer:
(d) 2

Question 9.
If |a|= 5, |b|= 13 and |a × b|= 25, find a.b
(a) ±10
(b) ±40
(c) ±60
(d) ±25
Answer:
(c) ±60

Question 10.
Find the value of λ so that the vectors $2i - 4\hat{j} + \hat{k}$ and $4i - 8\hat{j} + \lambda\hat{k}$ are parallel.
(a) -1
(b) 3
(c) -4
(d) 2
Answer:
(d) 2

Question 11.
If O is origin and C is the mid point of A(2, -1) and B(-4, 3), then the value of OC is
(a) $\hat{i} + \hat{j}$
(b) $\hat{i} - \hat{j}$
(c) $-\hat{i} + \hat{j}$
(d) $-\hat{i} - \hat{j}$
Answer:
(c) $-\hat{i} + \hat{j}$

Question 12.
The vectors AB = $3\hat{i} + 4\hat{k}$ and AC = $AC = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the side of a ΔABC. The length of the median through A is
(a) √18
(b) √72
(c) √33
(d) √288

Answer:
(c) √33

Question 13.
The summation of two unit vectors is a third unit vector, then the modulus of the difference of the unit vector is
(a) √3
(b) 1 – √3
(c) 1 + √3
(d) -√3
Answer:
(a) √3

Question 14.
Let $\vec{a}=\hat{i}+\hat{j}-\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\hat{c}$ be a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then $\hat{c}$ is

(a) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{6}}(2\hat{i}-\hat{j}+\hat{k})$

Answer:
(d) $\frac{1}{\sqrt{6}}(2\hat{i}-\hat{j}+\hat{k})$

Question 15.
If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a}+\vec{b}|<1$, then

(a) $\theta=\frac{\pi}{2}$ (b) $\theta<\frac{\pi}{3}$ (c)

$\pi \geq \theta > \frac{2\pi}{3}$ (d) $\frac{\pi}{3}<\theta<\frac{2\pi}{3}$

Answer:
(c) $\pi \geq \theta > \frac{2\pi}{3}$

Question 16.
The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is
(a) $\frac{2}{3}$
(b) $\frac{3}{2}$
(c) $\frac{5}{2}$
(d) $\frac{2}{5}$
Answer:
(a) $\frac{2}{3}$

Question 17.
The vectors from origin to the points A and B are a = $2\hat{i} - 3\hat{j} + 2\hat{k}$ and b = $2\hat{i} + 3\hat{j} + \hat{k}$, respectively then the area of triangle OAB is
(a) 340
(b) √25
(c) √229
(d) $\frac{1}{2}$ √229
Answer:
(d) $\frac{1}{2}$ √229

Question 18.
The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}, \hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if
(a) λ = -2
(b) λ = 0
(c) λ = 1
(d) λ = -1
Answer:
(a) λ = -2

Question 19.
If a, b, c are unit vectors such that a + b + c = 0, then the value of a.b + b.c + c.a is
(a) 1
(b) 3
(c) $-\frac{3}{2}$
(d) None of these
Answer:
(c) $-\frac{3}{2}$

Question 20.
If |a| = 4 and -3 ≤ λ ≤ 2, then the range of |λa| is
(a) [0, 8]

(b) [-12, 8]
(c) [0, 12]
(d) [8, 12]
Answer:
(c) [0, 12]

Question 21.
The number of vectors of unit length perpendicular to the vectors a = $2\hat{i} + \hat{j} + 2\hat{k}$ and b = $\hat{j} + \hat{k}$ is
(a) one
(b) two
(c) three
(d) infinite
Answer:
(b) two

Question 22.
Let a, b and c be vectors with magnitudes 3, 4 and 5 respectively and a + b + c = 0, then the values of a.b + b.c + c.a is
(a) 47
(b) 25
(c) 50
(d) -25
Answer:
(d) -25

Question 23.
If |a| = |b| = 1 and |a + b| = √3, then the value of (3a – 4b).(2a + 5b) is
(a) -21
(b) $-\frac{21}{2}$
(c) 21
(d) $\frac{21}{2}$
Answer:
(b) $-\frac{21}{2}$

Question 24.

The unit vector perpendicular to $\hat{i}-\hat{j}$ and coplanar with $\hat{i}+2\hat{j}$ and $2\hat{i}+3\hat{j}$ is

(a) $\frac{2\hat{i}-5\hat{j}}{\sqrt{29}}$ (b) $2\hat{i}+5\hat{j}$

(c) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (d) $\hat{i}+\hat{j}$

Answer:
(c) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

Question 25.
If |a – b| = |a| = |b| = 1, then the angle between a and b is
(a) $\frac{\pi}{3}$
(b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{2}$
(d) 0
Answer:
(a) $\frac{\pi}{3}$

Question 26.

$(\vec{a}.\hat{i})^2+(\vec{a}.\hat{j})^2+(\vec{a}.\hat{k})^2$ is equal to

(a) 1 (b) $\hat{a}$ (c) $-\hat{a}$ (d) $|\vec{a}|^2$

Answer:
(d) $|a|^2$

Question 27.
a, b, c are three vectors, such that a + b + c = 0, |a|= 1, |b|= 2, |c|= 3, then a.b + b.c + c is equal to
(a) 0
(b) -7
(c) 7
(d) 1
Answer:
(b) -7

Question 28.
If |a + b| = |a – b|, then angle between a and b is (a ≠ 0, b ≠ 0)

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{2}$
Answer:
(d) $\frac{\pi}{2}$

Question 29.
If a and b are two unit vectors inclined to x-axis at angles 30° and 120° respectively, then |a + b| equals
(a) $\sqrt{\frac{2}{3}}$
(b) √2
(c) √3
(d) 2
Answer:
(d) 2

Question 30.
If the angle between $\hat{i}+\hat{k}$ and $\hat{i}+\hat{j}+a\hat{k}$ is $\frac{\pi}{3}$, then the value of a is
(a) 0 or 2
(b) -4 or 0
(c) 0 or -3
(d) 2 or -2
Answer:
(b) -4 or 0

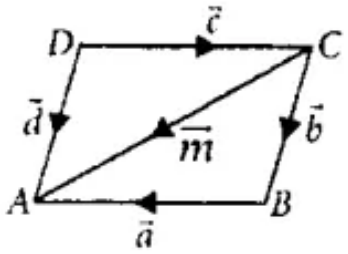
Question 31.
The length of longer diagronai of the parallelogram constructed on 5a + 2b and a – 3b. If it is given that
|a| = 2√2, |b| = 3 and angle between a and b is $\frac{\pi}{4}$, is
(a) 15
(b) √113
(c) √593
(d) √369
Answer:
(c) √593

Question 32.
If $\left(\frac{1}{2},\frac{1}{3},n\right)$ are the direction cosines of a line, then the value of n is
(a) $\frac{\sqrt{23}}{6}$

(b) $\frac{23}{6}$
(c) $\frac{2}{3}$
(d) $\frac{3}{2}$
Answer:
(a) $\frac{\sqrt{23}}{6}$

Question 33.
Find the magnitude of vector $3\hat{i} + 2\hat{j} + 12\hat{k}$.
(a) √157
(b) 4√11
(c) √213
(d) 9√3
Answer:
(a) √157

Direction (34 – 36): Study the given parallelogram and answer the following questions.



Question 34.
Which of the following represents equal vectors?
(a) a, c
(b) b, d
(c) b, c
(d) m, d
Answer:
(b) b, d

Question 35.
Which of the following represents collinear but not equal vectors?
(a) a, c
(b) b, d
(c) b, m
(d) Both (a) and (b)
Answer:
(a) a, c

Question 36.
Which of the following represents coinitial vector?
(a) c, d
(b) m, b
(c) b, d
(d) Both (a) and (b)
Answer:
(d) Both (a) and (b)

Question 37.
The unit vector in the direction of the sum of vectors
$\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{j}+4\hat{k}$ is

(a) $\frac{1}{5\sqrt{2}}(3\hat{i}+4\hat{j}+5\hat{k})$ (b) $\frac{1}{5\sqrt{2}}(3\hat{i}-4\hat{j}-5\hat{k})$

(c) $\frac{1}{2\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})$ (d) $\frac{1}{3\sqrt{2}}(-3\hat{k}+4\hat{i}+5\hat{j})$

Answer:
(a) $\frac{1}{5\sqrt{2}}(3\hat{i}+4\hat{j}+5\hat{k})$

Question 38.
The vectors $3\hat{i}+5\hat{j}+2\hat{k}, 2\hat{i}-3\hat{j}-5\hat{k}$ and $5\hat{i}+2\hat{j}-3\hat{k}$ form the sides of
(a) Isosceles triangle
(b) Right triangle
(c) Scalene triangle
(d) Equilaterala triangle
Answer:
(d) Equilaterala triangle

Question 39.
If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$, then

(a) $\alpha=1, \beta=-1$ (b) $\alpha=1, \beta=\pm1$
(c) $\alpha=-1, \beta=\pm1$ (d) $\alpha=\pm1, \beta=1$

Answer:
(d) $\alpha=\pm1, \beta=1$

Question 40.
The vectors $a = x\hat{i} - 2\hat{j} + 5\hat{k}$ and $b = \hat{i} + y\hat{j} - z\hat{k}$ are collinear, if
(a) x =1, y = -2, z = -5
(b) x= 1.2, y = -4, z = -10
(c) x = -1/2, y = 4, z = 10
(d) All of these
Answer:
(d) All of these

Question 41.
The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\hat{i} + 2\hat{k}$. The value of x is
(a) $\left\{-\frac{2}{3}, 2\right\}$
(b) $\left\{\frac{1}{3}, 2\right\}$
(c) $\left\{\frac{2}{3}, 0\right\}$
(d) {2, 7}
Answer:
(a) $\left\{-\frac{2}{3}, 2\right\}$

Question 42.
If a + b + c = 0, then a × b =
(a) c × a
(b) b × c
(c) 0
(d) Both (a) and (b)
Answer:
(d) Both (a) and (b)

Question 43.
If a is perpendicular to b and c, |a| = 2, |b| = 3, |c| = 4 and the angle between b and c is $\frac{2\pi}{3}$, |abc| is equal to
(a) 4√3
(b) 6√3
(c) 12√3
(d) 18√3
Answer:
(c) 12√3

Question 44.

If $\vec{b}$ and $\vec{c}$ are any two non-collinear unit vectors and $\vec{a}$ is any vector, then

$(\vec{a}.\vec{b})\vec{b}+(\vec{a}.\vec{c})\vec{c}+\frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}.(\vec{b}\times\vec{c})$ **is equal to**

(a) $\vec{0}$ (b) $\vec{a}$ (c) $\vec{b}$ (d) $\vec{c}$

Answer:
(b) a

Question 45.

Let $\vec{a}=\hat{i}-\hat{k},\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ **and** $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. **Then,** $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ **depends on**

(a) neither x nor y (b) both x and y
(c) only x (d) only y

Answer:
(a) neither x nor y

Question 46.
If a, b, c are three non-coplanar vectors, then (a + b + c).[(a + b) × (a + c)] is
(a) 0
(b) 2[abc]
(c) -[abc]
(d) [abc]
Answer:
(c) -[abc]

Question 47.
If u, v and w are three non-coplanar vectors, then (u + v – w).[(u – v) × (v – w)] equals
(a) 0
(b) u.v × w
(c) u.w × v
(d) 3u.v × w
Answer:
(b) u.v × w

Question 48.
If unit vector c makes an angle $\frac{\pi}{3}$ with $\hat{i}\times\hat{j}$, then minimum and maximum values of $(\hat{i}\times\hat{j})\cdot c$

respectively are
(a) 0, $\frac{\sqrt{3}}{2}$
(b) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$
(c) -1, $\frac{\sqrt{3}}{2}$
(d) None of these
Answer:
(b) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$

Question 49.
The volume of the tetrahedron whose conterminous edges are $\hat{j}+\hat{k}, \hat{i}+\hat{k}, i+\hat{j}$ is
(a) $\frac{1}{6}$ cu. unit
(b) $\frac{1}{3}$ cu. unit
(c) $\frac{1}{2}$ cu. unit
(d) $\frac{2}{3}$ cu. unit
Answer:
(b) $\frac{1}{3}$ cu. unit

Question 50.
If the vectors $2\hat{i}-3\hat{j}, i+\hat{j}-\hat{k}$ and $3\hat{i}-\hat{k}$ form three concurrent edges of a parallelopiped, then the volume of the parallelopiped is
(a) 8
(b) 10
(c) 4
(d) 14
Answer:
(c) 4

Question 51.
The volume of the parallelopiped whose edges are represented by $-12\hat{i}+\alpha\hat{k}, 3j-\hat{k}$ and $2\hat{i}+j-15\hat{k}$ is 546 cu. units. Then α =
(a) 3
(b) 2
(c) -3
(d) -2
Answer:
(c) -3

Question 52.

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - 4\hat{k}, \vec{c} = \hat{i} + \lambda\hat{j} + 6\hat{k}$ are coplanar, then the value of λ is

(a) $\frac{5}{2}$ (b) $\frac{3}{5}$

(c) $\frac{7}{3}$ (d) None of these

Answer:
(d) None of these

Question 53.

The value of λ, for which the four points $2\hat{i} + 3\hat{j} - \hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}, 3\hat{i} + 4\hat{j} - 2\hat{k}, \hat{i} - \lambda\hat{j} + 6\hat{k}$ are coplanar, is

(a) –2 (b) 8 (c) 6 (d) 0

Answer:
(a) -2

Question 54.

The vectors $x\hat{i} + (x+1)\hat{j} + (x+2)\hat{k}$, $(x+3)\hat{i} + (x+4)\hat{j} + (x+5)\hat{k}$ and $(x+6)\hat{i} + (x+7)\hat{j} + (x+8)\hat{k}$ are coplanar for

(a) all values of x (b) $x < 0$

(c) $x \le 0$ (d) None of these

Answer:
(a) all values of x

Question 55.

If the vectors $\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}, \lambda\hat{i} - \hat{j} + 2\hat{k}$ are coplanar, then the value of λ is equal to

(a) 0
(b) 1
(c) 2
(d) 3

Answer:
(a) 0

Question 56.
Find the value of λ if the vectors, a = $2\hat{i} - \hat{j} + \hat{k}$, b = $\hat{i} + 2\hat{j} - 3\hat{k}$ and c = $3\hat{i} - \lambda\hat{j} + 5\hat{k}$ are coplanar.
(a) 4
(b) -2
(c) -6
(d) 5
Answer:
(a) 4

Question 57.
If a, b, c are unit vectors, then |a – b| + |b – c| + |c – a| does not exceed
(a) 4
(b) 9
(c) 8
(d) 6
Answer:
(b) 9

Question 58.
Find the value of λ so that the vectors $2\hat{i} - 4\hat{j} + \hat{k}$ and $4\hat{i} - 8\hat{j} + \lambda\hat{k}$ are perpendicular.
(a) -15
(b) 10
(c) -40
(d) 20
Answer:
(c) -40

Question 59.
The dot product of a vector with the vectors $\hat{i} + \hat{j} - 3\hat{k}, \hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.
(a) $\hat{i} + 2\hat{j} + \hat{k}$
(b) $-\hat{i} + 3\hat{j} - 2\hat{k}$
(c) $\hat{i} + 2\hat{j} + 3\hat{k}$
(d) $\hat{i} - 3\hat{j} - 3\hat{k}$
Answer:
(a) $\hat{i} + 2\hat{j} + \hat{k}$

Question 60.
If a, b, c are three mutually perpendicular vectors of equal magnitude, find the angle between a and a + b + c.
(a) $\cos^{-1}(1/\sqrt{3})$

(b) $\cos^{-1}(1/2\sqrt{2})$
(c) $\cos^{-1}(1/3\sqrt{3})$
(d) $\cos^{-1}(1/2\sqrt{3})$
Answer:
(a) $\cos^{-1}(1/\sqrt{3})$